United Kingdom Mathematics Trust

British Mathematical Olympiad

Round 1 : Thursday, 2 December 2010

Time allowed $3\frac{1}{2}$ hours.

Instructions

- *Full written solutions – not just answers – are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt. Do not hand in rough work.*
- *One* complete *solution will gain more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all the problems.*
- *Each question carries 10 marks. However, earlier questions tend to be easier. In general you are advised to concentrate on these problems first.*
- *The use of rulers and compasses is allowed, but calculators and protractors are forbidden.*
- *Start each question on a fresh sheet of paper. Write on one side of the paper only. On each sheet of working write the number of the question in the top* left *hand corner and your name, initials and school in the top* right *hand corner.*
- *Complete the cover sheet provided and attach it to the front of your script, followed by your solutions in question number order.*
- *Staple all the pages neatly together in the top* left *hand corner.*
- *To accommodate candidates sitting in other time-zones, please do not discuss the paper on the internet until 8am on Friday 3 December GMT.*

Do not turn over until **told to do so.**

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1. One number is removed from the set of integers from 1 to n. The average of the remaining numbers is $40\frac{3}{4}$. Which integer was removed?

2. Let s be an integer greater than 6. A solid cube of side s has a square hole of side $x < 6$ drilled directly through from one face to the opposite face (so the drill removes a cuboid). The volume of the remaining solid is numerically equal to the total surface area of the remaining solid. Determine all possible integer values of x.

3. Let ABC be a triangle with $\angle CAB$ a right-angle. The point L lies on the side BC between B and C. The circle ABL meets the line AC again at M and the circle CAL meets the line AB again at N. Prove that L, M and N lie on a straight line.

4. Isaac has a large supply of counters, and places one in each of the 1×1 squares of an 8×8 chessboard. Each counter is either red, white or blue. A particular pattern of coloured counters is called an *arrangement*. Determine whether there are more arrangements which contain an even number of red counters or more arrangements which contain an odd number of red counters. *Note that* 0 *is an even number.*

5. Circles S_1 and S_2 meet at L and M. Let P be a point on S_2. Let PL and PM meet S_1 again at Q and R respectively. The lines QM and RL meet at K. Show that, as P varies on S_2, K lies on a fixed circle.

6. Let a, b and c be the lengths of the sides of a triangle. Suppose that $ab + bc + ca = 1$. Show that $(a + 1)(b + 1)(c + 1) < 4$.